Finding ALL Solutions (3.10) HW

Solve over the interval $0 \leq \theta < 2\pi$. Then, write an expression that represents ALL of the solutions.

1. $\sin\theta = \frac{\sqrt{3}}{2}$

2. $\cot\theta = -\frac{\sqrt{3}}{3}$

3. $\sec\theta = -1$

4. $\tan\theta + 1 = 0$

5. $2\csc\theta + 4 = 0$

6. $\sqrt{3}\tan\theta = 3$

7. $(2\cos\theta - 1)(\cos\theta) = 0$?

8. $(\sin\theta + 1)\left(\sqrt{2}\sin\theta - 1\right) = 0$

9. $(\sec\theta + 1)(\sec\theta - 2) = 0$

10. $(2\sin\theta + 1)(\csc\theta) = 0$

11. $(\tan\theta)^2 = 3$

12. $2(\sin\theta)^2 - (\sin\theta) = 0$ (hint: factor)

Remember when we used to have speed trig ratio quizzes? Those were the days...Let's do another one! For old time's sake. This time, any angle in the interval $0 \leq \theta \leq 2\pi$ could show up. And any of the six trig functions are fair game. Scan/click/tap the QR code to the right to practice. After time runs out, this website gives an explanation for how to think about each problem with reference angles.

Finding ALL Solutions HW Answers

1. We are looking for a first or second quadrant angle with a reference angle of $\frac{\pi}{3}$
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{\pi}{3}$ or $\theta = \frac{2\pi}{3}$
   All solutions are of the form $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{2\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$
2. We are looking for a second or fourth quadrant angle with a reference angle of $\frac{\pi}{3}$
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$
   All solutions are of the form $\theta = \frac{2\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$
3. The solution to this will be quadrantal
   The only solutions in $0 \leq \theta < 2\pi$ is $\theta = \pi$
   All solutions are of the form $\theta = \pi + 2\pi k$ where $k \in \mathbb{Z}$
4. $\tan\theta = -1$
   We are looking for a second or fourth quadrant angle with a reference angle of $\frac{\pi}{4}$
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{3\pi}{4}$ or $\theta = \frac{7\pi}{4}$
   All solutions are of the form $\theta = \frac{3\pi}{4} + 2\pi k$ or $\theta = \frac{7\pi}{4} + 2\pi k$ where $k \in \mathbb{Z}$
   This can be stated more concisely as $\theta = \frac{3\pi}{4} + \pi k$ where $k \in \mathbb{Z}$
5. $\csc\theta = -2$
   We are looking for a third or fourth quadrant angle with a reference angle of $\frac{\pi}{6}$
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$
   All solutions are of the form $\theta = \frac{7\pi}{6} + 2\pi k$ or $\theta = \frac{11\pi}{6} + 2\pi k$ where $k \in \mathbb{Z}$
6. $\tan\theta = \sqrt{3}$
   We are looking for a first or third quadrant angle with a reference angle of $\frac{\pi}{3}$
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{\pi}{3}$ or $\theta = \frac{4\pi}{3}$
   All solutions are of the form $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{4\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$
   This can be stated more concisely as $\theta = \frac{\pi}{3} + \pi k$ where $k \in \mathbb{Z}$
7. $2\cos\theta - 1 = 0$ or $\cos\theta = 0$
   $\cos\theta = \frac{1}{2}$ or $\cos\theta = 0$
   θ is a first or fourth quadrant angle with reference angle $\frac{\pi}{3}$ or θ is a quadrant angle where the x-value on the unit circle is 0.
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$ or $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$
   All solutions are of the form $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$ or $\theta = \frac{\pi}{2} + 2\pi k$ or $\theta = \frac{3\pi}{2} + 2\pi k$ where k is an integer.
   This can be stated more concisely as $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$ or $\theta = \frac{\pi}{2} + \pi k$ where k is an integer.
8. $\sin\theta + 1 = 0$ or $\sqrt{2}\sin\theta - 1 = 0$
   $\sin\theta = -1$ or $\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   θ is a quadrantal angle where the y-value on the unit circle is -1 or θ is a first or second quadrant angle with reference angle $\frac{\pi}{4}$.
   The only solutions in $0 \leq \theta < 2\pi$ are $\theta = \frac{3\pi}{2}$ or $\theta = \frac{\pi}{4}$ or $\theta = \frac{3\pi}{4}$
   All solutions are of the form $\theta = \frac{3\pi}{2} + 2\pi k$ or $\theta = \frac{\pi}{4} + 2\pi k$ or $\theta = \frac{3\pi}{4} + 2\pi k$ where $k \in \mathbb{Z}$

9. $\sec\theta + 1 = 0$ or $\sec\theta - 2 = 0$
   $\sec\theta = -1$ or $\sec\theta = 2$
   θ is a quadrantal angle where $\frac{1}{x}$ on the unit circle is -1 or θ is a first or fourth quadrant angle with a reference angle of $\frac{\pi}{3}$.
   The only solutions in $0 \le \theta < 2\pi$ are $\theta = \pi$ or $\theta = \frac{\pi}{3}$ or $\theta = \frac{5\pi}{3}$
   All solutions are of the form $\theta = \pi + 2\pi k$ or $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$.
   This can be expressed more efficiently as $\theta = \frac{\pi}{3} + \frac{2\pi}{3}k$ where $k \in \mathbb{Z}$.
10. $2\sin\theta + 1 = 0$ or $\csc\theta = 0$
    $\sin\theta = -\frac{1}{2}$ or θ corresponds with a point on the unit circle where $\frac{1}{y} = 0$ (which is impossible).
    θ is a third or fourth quadrant angle with a reference angle of $\frac{\pi}{6}$.
    The only solutions in $0 \le \theta < 2\pi$ are $\theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$.
    All solutions are of the form $\theta = \frac{7\pi}{6} + 2\pi k$ or $\theta = \frac{11\pi}{6} + 2\pi k$ for $k \in \mathbb{Z}$.
11. $\tan\theta = \sqrt{3}$ or $\tan\theta = -\sqrt{3}$
    θ is first or third quadrant angle with a reference angle of $\frac{\pi}{3}$ or θ is a second or fourth quadrant angle with a reference angle of $\frac{\pi}{3}$.
    The only solutions in $0 \le \theta < 2\pi$ are $\theta = \frac{\pi}{3}$ or $\theta = \frac{4\pi}{3}$ or $\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$
    All solutions are of the form $\theta = \frac{\pi}{3} + 2\pi k$ or $\theta = \frac{4\pi}{3} + 2\pi k$ or $\theta = \frac{2\pi}{3} + 2\pi k$ or $\theta = \frac{5\pi}{3} + 2\pi k$ where $k \in \mathbb{Z}$.
    This can be expressed more efficiently as $\theta = \frac{\pi}{3} + \pi k$ or $\theta = \frac{2\pi}{3} + \pi k$ where $k \in \mathbb{Z}$.
12. $(\sin\theta)(2\sin\theta - 1) = 0$
    $\sin\theta = 0$ or $2\sin\theta - 1 = 0$
    $\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$
    θ corresponds with a point on the unit circle with a y-value of 0 or θ is in the first or second quadrant with a reference angle of $\frac{\pi}{6}$.
    The only solutions in the interval $0 \le \theta < 2\pi$ are $\theta = 0$ or $\theta = \pi$ or $\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$.
    All solutions are of the form $\theta = 2\pi k$ or $\theta = \pi + 2\pi k$ or $\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{5\pi}{6} + 2\pi k$ where $k \in \mathbb{Z}$.
    This can be expressed more efficiently as $\theta = \pi k$ or $\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{5\pi}{6} + 2\pi k$ where $k \in \mathbb{Z}$.